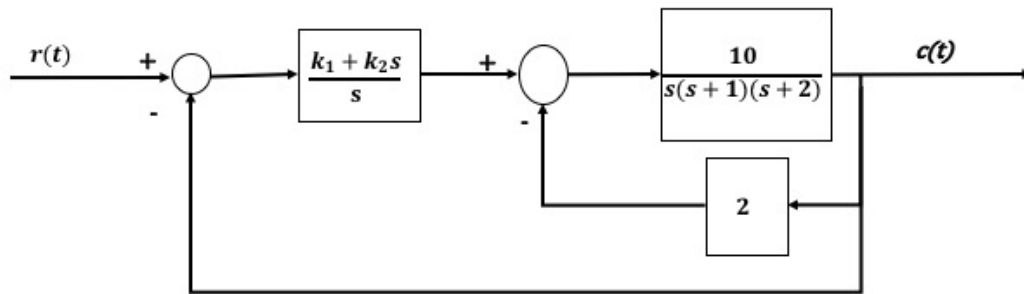


Question 1 (20 points)

For the shown block diagram, assuming $k_1 = 1$ and $k_2 = 0$, find:

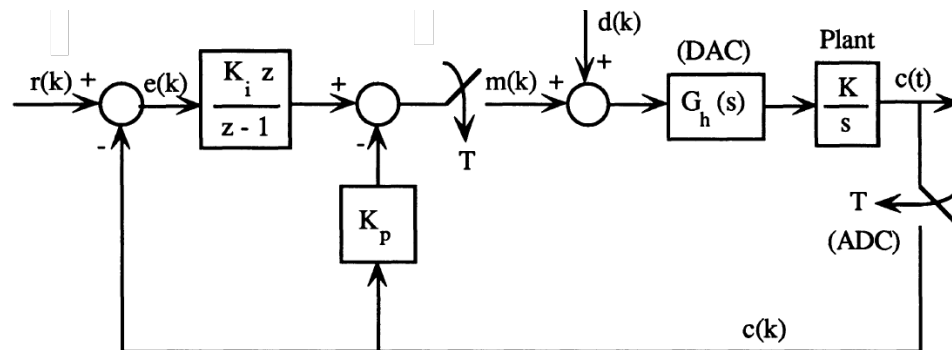
- The closed-loop transfer function (10 points).
- The steady-state error value if the input is a unit step function (10 points)



Question 2 (25 points)

For the sampled block diagram below, $G_h(s) = \frac{1 - e^{-sT}}{s}$, determine:

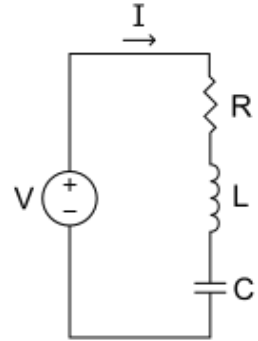
- The closed-loop transfer function $\frac{C(z)}{R(z)}$ [Hint: assume zero disturbance] (5 points)
- The output-to-disturbance transfer function $\frac{C(z)}{D(z)}$ [Hint: Assume zero input] (5 points)
- The unit-step response assuming $K = 20, K_p = 5, K_i = 1.25, T = 0.005$ (5 points)
- The stability constraints on K_p and K_i . Plot on a $K_p - K_i$ plane (10 points)



Question 3 (25 points)

For the RLC circuit shown, the system equations are as follows:

$$\begin{aligned}V_R &= RI \\V_L &= L\dot{I} \\V_C &= \frac{1}{C} \int I dt \\V &= V_R + V_L + V_C\end{aligned}$$



If the system input is V and the outputs are V_R and V_C :

- Find the state space model of the system (10 points)
- Find the closed-loop transfer function(s) of the system (5 points)
- Determine if the system is controllable (5 points)
- Determine if the system is observable (5 points)

Question 4 (45 points)

Design the following filters:

- 2nd order Binomial low-pass filter with cut-off frequency of 50 r/s (5 points)
- 3rd order Butterworth high-pass filter with cut-off frequency of 50 r/s (10 points)
- 3rd order Butterworth digital high-pass filter with cut-off frequency of 50 r/s and sampling frequency of 60Hz. Use the bilinear transformation method. (10 points)
- 4th order Bessel band-stop filter with cut-off frequency of 50 r/s and bandwidth of 10 r/s (20 points)

Good Luck

Stability of Discrete Systems

First Order (n=1): $F(z) = a_1 z + a_0 = 0$, $a_1 > 0$

$$\text{Stability constraints: } \left| \frac{a_0}{a_1} \right| < 1$$

Second Order (n=2): $F(z) = a_2 z^2 + a_1 z + a_0 = 0$, $a_2 > 0$

$$\begin{aligned} \text{Stability constraints: } & a_2 + a_1 + a_0 > 0 \\ & a_2 - a_1 + a_0 > 0 \\ & |a_0| < a_2 \end{aligned}$$

Third Order (n=3): $F(z) = a_3 z^3 + a_2 z^2 + a_1 z + a_0 = 0$, $a_3 > 0$

$$\begin{aligned} \text{Stability constraints: } & a_3 + a_2 + a_1 + a_0 > 0 \\ & a_3 - a_2 + a_1 - a_0 > 0 \\ & |a_0| < a_3 \\ & |a_0^2 - a_3^2| > |a_0 a_2 - a_1 a_3| \end{aligned}$$

State Space Model

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

Controllability condition:

$$\text{Rank}[B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B] = n$$

Observability condition:

$$\text{Rank} \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$$

Filter Design

Binomial LPF:

$$G(s) = \frac{\omega_c^n}{(s + \omega_c)^n}$$

Butterworth LPF:

$$G(s) = \frac{\omega_c^n}{\prod_{k=0}^{n-1} (s - s_k)}$$

$$s_k = \omega_c e^{\frac{j\pi}{2}(1 + \frac{2k+1}{n})}$$

Bessel LPF:

$$G(s) = \frac{\theta_n(0)}{\theta_n\left(\frac{s}{\omega_c}\right)}$$

$$\theta_n(x) = \sum_{k=0}^n a_k x^k$$

$$\theta_n(0) = \lim_{x \rightarrow 0} \theta_n(x)$$

$$a_k = \frac{(2n-k)!}{2^{n-k} k! (n-k)!}$$

HPF, BPF, BSF:

$$G_{HPF}(s) = G_{LPF}\left(\frac{1}{s}\right)$$

$$G_{BPF}(s) = G_{LPF}\left(\frac{s^2 + \omega_c^2}{Bs}\right)$$

$$G_{BSF}(s) = G_{HPF}\left(\frac{s^2 + \omega_c^2}{Bs}\right)$$

$$\omega_c = \omega_1 \omega_2, B = (\omega_2 - \omega_1)$$

Bilinear Transformation Method:

$$G(z) = G(s) \Big|_{s=\frac{z-1}{z+1}}$$

$$\omega_c = \tan\left(\frac{\omega_d T}{2}\right)$$

Laplace and Z-Transform

$x(t)$	$x(kT)$	$X(s)$	$X(z)$
$\delta(t)$	$\delta(k)$	1	1
$u(t)$	$u(k)$	$\frac{1}{s}$	$\frac{z}{z-1}$
t	kT	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
e^{-at}	e^{-akT}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
te^{-at}	kTe^{-akT}	$\frac{1}{(s+a)^2}$	$\frac{z}{(z-e^{-aT})^2}$
$e^{-at}\cos(\omega t)$	$e^{-akT}\sin(\omega kT)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2 - ze^{-at}\cos(\omega t)}{z^2 - 2ze^{-aT}\cos(\omega t) + e^{-2aT}}$
$e^{-at}\sin(\omega t)$	$e^{-akT}\sin(\omega kT)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-at}\sin(\omega t)}{z^2 - 2ze^{-aT}\cos(\omega t) + e^{-2aT}}$
		$(1 - e^{-sT})M(s)$	$(1 - z^{-1})M(z)$